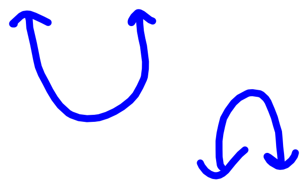
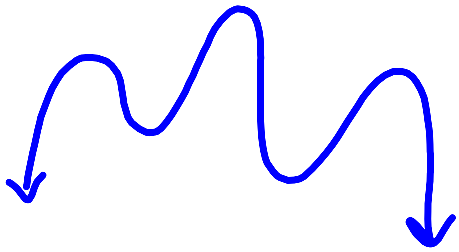
# Concavity

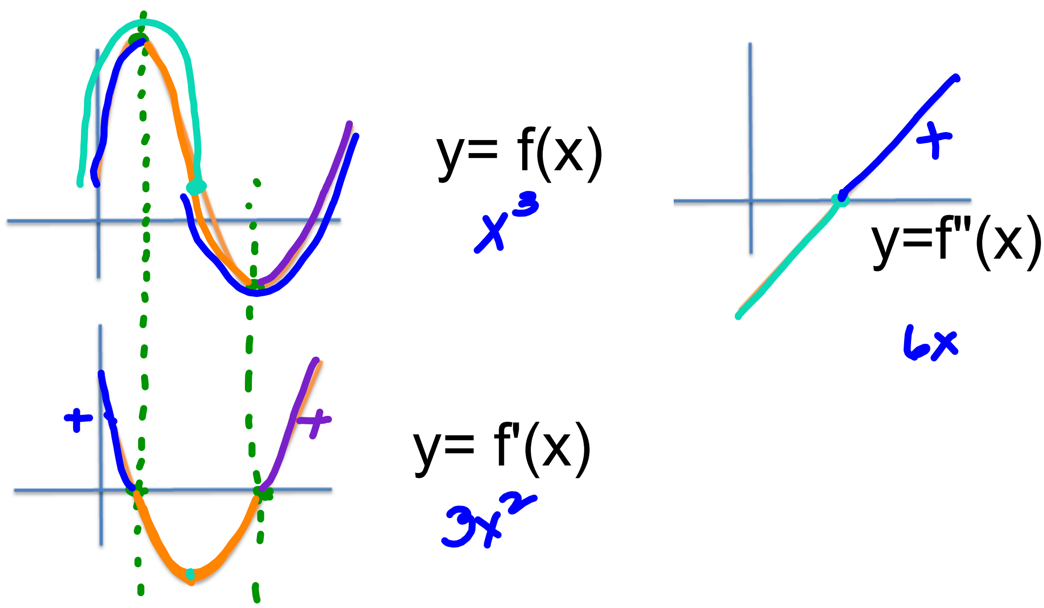


## and the Second Derivative Test



# Concavity:

- *f* is differentiable on an open interval
- The graph of *f* is concave upward on *I* when $f'$ is increasing on the interval.
- The graph of *f* is concave downward on *I* when $f'$ is decreasing on the interval


y= f(x)
x³
+
+
y= f'(x)
3x²
+
y=f"(x)
6x

# Test for Concavity

1. If $f''(x) > 0$ for all x in the interval, then the graph of *f* is **concave upward** on the interval.

2. If $f''(x) < 0$ for all x in the interval, then the graph of *f* is **concave downward** on the interval.

Determine the open intervals on which the graph of

$$f(x) = \frac{6}{x^2 + 3}$$

is concave upward or downward.

$f'(x) = \frac{-12x}{(x^2+3)^2}$

$f''(x) = \frac{36(x^2-1)}{(x^2+3)^3}$

① $f'(x)$
② $f''(x)$
③ CN $f''(x) = 0$

$f''(x) = \frac{36(x^2-1)}{(x^2+3)^3} = 0$

$x = \pm 1$

| intervals | $(-\infty, -1)$ | $(-1, 1)$ | $(1, \infty)$ |
|---|---|---|---|
| Test values | $-2$ | $0$ | $2$ |
| $f''(c)$ | $+$ | $-$ | $+$ |
| | up | down | up |

Determine the open intervals on which the graph of $f(x)=\dfrac{x^2+1}{x^2-4}$ is concave up or down

$f'(x)=\dfrac{-10x}{(x^2-4)^2}$

$f''(x)=\dfrac{10(3x^2+4)}{(x^2-4)^3}$

$\dfrac{10(3x^2+4)}{(x^2-4)^3}=0$ No CN

$(x^2-4)^3=0$ $x=\pm 2$ discontinuities

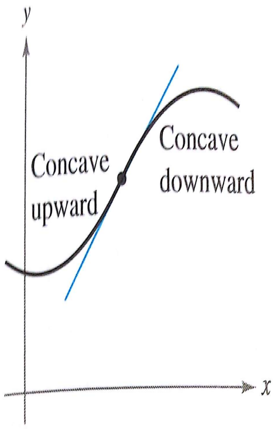
| | | | |
|---|---|---|---|
| | $(-\infty,-2)$ | $(-2,2)$ | $(2,\infty)$ |
| | $-11$ | $\frac{3}{4}$ | $\frac{67}{3}$ |
| | | | |
| | + | − | + |

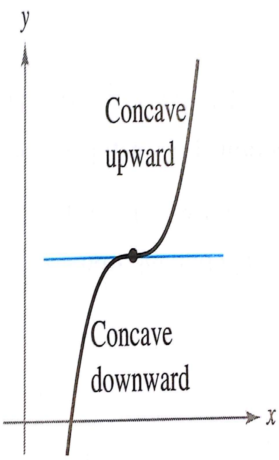
Points of inflection

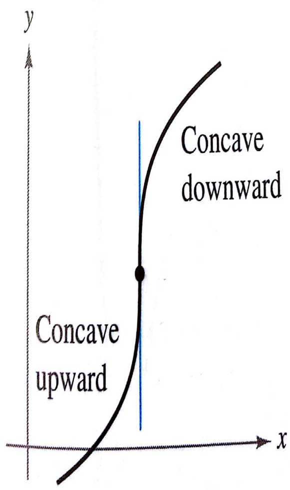
If *f* is continuous on an open interval and

- the concavity changes from up to down or down to up
- c has a tangent line at that point.

**Then there is a point of inflection at point c.**



If (c, f(c) ) is a point of inflection on the graph of *f*, then either *f''(c)* =0 or *f''(c)* DNE at x=c

Determine the points of inflection and discuss the concavity of the graph of

$$f\left(x\right)=x^4-4x^3$$

| | | | |
|---|---|---|---|
| | | | |
| | | | |
| | | | |

The Second Derivative Test

$\begin{cases} f \text{ is a function} \\ f'(c)=0 \\ f'' \text{ exists on an open interval containing } c \end{cases}$

**1. If f"(c) >0, then f has a relative min at c**

**2. If f"(c) <0, then f has a relative max at c**

If f"(c)=0, the test fails. In this case, you can use the first derivative test.

Find the relative extrema of

$$f(x) = -3x^5 + 5x^3$$

| | | | |
|---|---|---|---|
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| | | | |